A free-space spectral Poisson solver for the Nirvana-III code

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Many astrophysical phenomena involve accounting for the gravity of dilute gas, known as self-gravity (SG). These include the core collapse of a molecular cloud (Mauxion et al. 2024), episodic outbursts in FU Orionis (Armitage et al. 2001; Vorobyov & Basu 2006), and accretion and angular momentum transport in young protoplanetary disks, which can lead to fragmentation with efficient cooling (Kratter & Lodato 2016). Recent studies have even shown that gravitational instability can amplify weak magnetic fields through a process known as the gravitoturbulent dynamo process (Riols & Latter 2019). Thus, accurately and efficiently solving the Poisson equation is crucial for studying these phenomena. A standard method for Cartesian uniform grids uses iterative multigrid methods (Ziegler 2004), but this requires precise evaluation of the SG potential at domain boundaries, often necessitating a multipole expansion. A more suitable, albeit complex, method is the screening, or James, method (James 1977; Gressel & Ziegler 2024), which estimates the exact potential at domain boundaries.

Numerical computation of the gravitational potential can also be performed using spectral methods, which involve solving the Poisson equation in Fourier space and then using an inverse transform to obtain the potential in real space. Their advantage lies in their accuracy and efficiency, with a numerical complexity of $N \log(N)$, making them suitable for large scale problems. However, a major drawback is the requirement for periodicity in all directions, which is unrealistic and necessitates that the volume integral of the source term of the Poisson equation vanishes (Binney & Tremaine 2008; Mandal et al. 2023). Interestingly, a method overlooked by the astrophysical community but well-known in condensed matter and plasma physics overcomes these difficulties by modifying the Green's function (Vico et al. 2016) (hereafter VGF method). Under this formalism, the Green's function in Fourier space admits an analytical form and is regularized at the singularity, making it suitable for Fast Fourier Transform (FFT) methods. Benchmarks have shown that the relative errors of this method reach machine accuracy with a modest number of points, defying any fixed order of convergence (Zou et al. 2021; Mayani et al. 2024). However, this method is not yet adapted to the shearing box approximation (Goldreich & Lynden-Bell 1965), which requires two periodic boundary conditions and one unbound boundary condition.

In this talk, I propose presenting a new spectral solver for the Poisson equation designed for Cartesian geometry in the Finite Volume Code Nirvana-iii (Rendon Restrepo & Gressel in prep.). Specifically, I will detail the implementation of the VGF method for 3D unbound problems and introduce a new analytical Green's function for 3D shearing box simulations. The focus will be on the spectral accuracy of the method and the performance of the new solver, which utilizes the pencil decomposition of the P3DFFT library (Pekurovsky 2012).

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